End Field Modelling

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Why

- Want to examine multiple designs
- Can’t re-design magnets each time you make a lattice change
- Need good starting point to judge nonlinearities
  - Coming from end fields
  - Chromatic behavior
  - Dynamic aperture

Tracking Through Magnet Ends

- Compute result to first order in body field strenght
  - Can be computed independent of end shape
  - Arbitrary order in transverse variables
  - Limit as end length goes to zero
- Hamiltonian $H_p - H_q$
  - H_p independent of field
  - H_q linear in field
- Write map in Lie form $e^{:f:}$

$$f(s) = \sum_{k=1} f_k(s)$$

$$f_1(s) = \int^s H_q(\bar{s})\, d\bar{s} \qquad f_{n+1}(s) = \int^s [H_p, f_n(\bar{s})]\, d\bar{s}$$

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Tracking Through Magnet Ends (cont.)

Muon Collaboration

- If $\mathcal{S}_L(s)$ is a function going from 0 to 1 in length L, $L \to 0$,

$$\int_{-L/2}^{L/2} ds_1 \int_{-L/2}^{s_1} ds_2 \cdots \int_{-L/2}^{s_{n-1}} ds_n \, \mathcal{S}_L^{(k)}(s_n) = \delta_{kn}$$

- Thus f_k picks off terms proportional to the kth derivative of the field at the end
- Result is that f_{n+1} has larger transverse order than f_n

End Field Model

- Make some assumption on behavior of field at ends
  - Rate and form of falloff
  - Symmetry
- Types of end symmetry
  - Midplane: form of field in midplane is given: $B_y(x, 0, s)$
  - Multipole: in polar coordinates, B_r and B_ϕ in polar coordinates are of the form $f(r, s)\cos(m\phi)$
    - ⋆ Specify coefficient of $r^{m-1}\cos(m\phi)$
- These assumptions give different answers
  - Answers are the same if there is no s dependence
  - Which symmetry to choose depends on magnet construction
  - Could be other symmetries